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Dimensional reduction of the Abelian Higgs Carroll–Field–Jackiw model

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Abstract. Taking as a starting point a Lorentz non-invariant abelian Higgs model defined in $1+3$ dimensions, we carry out its dimensional reduction to $D = 1+2$, obtaining a new planar model composed by a Maxwell–Chern–Simons–Proca gauge sector, a massive scalar sector, and a mixing term (involving the fixed background v^{μ}) that imposes the Lorentz violation to the reduced model. The propagators of the scalar and massive gauge field are evaluated and the corresponding dispersion relations determined. Based on the poles of the propagators, a causality and unitarity analysis is carried out at tree level. We then show that the model is totally causal, stable and unitary.

1 Introduction

The point of view that some quantum field theories could be effective models originating from more fundamental theories has been enhanced with the advent of supersymmetry and supergravity, and more recently, with superstrings and branes. In the end of the 90s, some works [1] have demonstrated that a spontaneous violation of Lorentz symmetry can take place in the context of string theories. Some time later, the spontaneous violation of CPT and Lorentz symmetries was adopted as a possibility to define some CPT and Lorentz violating models which can be taken as the low-energy limit of an extension of the standard model defined at the Planck scale [2]. This master model undergoes a spontaneous symmetry breaking, generating an effective action that incorporates CPT and Lorentz violation and keeps unaffected the $SU(3) \times SU(2) \times U(1)$ gauge structure of the underlying theory. The Lorentz violation takes place at the level of particle transformations, whereas at the level of observer rotations and boosts the effective model remains Lorentz invariant. Such a difference comes from the role played by the CPT violating background term, v_{μ}, seen as a 4-vector under an observer Lorentz transformation and as a set of four scalars in a particle frame. Moreover, the Lorentz covariance is maintained as a feature of the underlying extended model, a consequence of spontaneous character of the symmetry breaking. This fact is of relevance in the sense that it indicates that the effective model may preserve some properties of the original theory, like causality and stability. Although Lorentz symmetry is closely connected to stability and causality in modern field theories, a model endowed with the latter properties in the absence of the former should be in principle acceptable and meaningful on physical grounds.

Lorentz violating theories have been in focus of recent and intensive investigation. Such models have been presently adopted as an attempt to explain the observation of ultra-high energy cosmic rays with energies beyond the Greisen–Zatsepin–Kuzmin (GZK) cutoff ($E_{\mathrm{GZK}} \simeq 4.10^{19}\,\mathrm{eV}$) [3,4], once such a kind of observation could be potentially taken as one piece of evidence of Lorentz violation. The rich phenomenology of fundamental particles has also been considered as a natural environment for the search for indications of breaking of these symmetries [3,5], indicating possible limitations associated with such a violation. Another point of interest refers to the issue of space-time varying coupling constants [6], which has been reassessed in the light of Lorentz violating theories, with interesting connections with the construction of supergravity models. Moreover, measurements of radio emission from distant galaxies and quasars put in evidence that the polarizations vectors of the radiation emitted are

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not randomly oriented as naturally expected. This peculiar phenomenon suggests that the space-time intervening between the source and observer may be exhibiting some sort of optical activity (birefringence), whose origin is unknown [8].

The pure gauge sector of the Lorentz violating low-energy effective model is composed basically by two types of terms with respect to CTP symmetry:
(i) the even CPT term, $k_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta}$, where the coupling $k_{\alpha\beta\gamma\delta}$ appears as a double traceless tensor with the same symmetries of the Riemann tensor, and $F^{\gamma\delta}$ is the field strength;
(ii) the odd CPT term, $\epsilon_{\mu\nu\kappa\lambda}v^{\mu}A^{\nu}F^{\kappa\lambda}$, where $\epsilon_{\mu\nu\kappa\lambda}$ is the 4-dimensional Levi-Civita symbol and v^{μ} is a fixed 4-vector acting as a background. This odd CPT term (a Chern–Simons-like mass term) was first considered in the context of a classical electrodynamics by Carroll–Field–Jackiw [7], setting up a simple way to realize the CPT and Lorentz breakings in the framework of the Maxwell theory. In spite of predicting several interesting new properties and phenomenology, the Carroll–Field–Jackiw (CFJ) model is plagued with some serious problems, like the absence of stability and causality in the case of a purely timelike background, $v_{\mu} = (v_0, 0)$. Even so, this theory has been the object of much attention in several different aspects, like the following ones:
(i) the birefringence (optical activity of the vacuum), induced by the fixed background [7,8],
(ii) the investigation of radiative corrections [9],
(iii) the consideration of spontaneous breaking of $U(1)$-symmetry in this framework [10],
(iv) the search for a supersymmetric Lorentz violating extension model [11],
(v) the study of vacuum Cerenkov radiation [12], the photon decay process [13], and the development of CFJ electrodynamics in a pre-metric framework [14].

The quest for a Lorentz violating model able to preserve the algebra of supersymmetry (SUSY) was first addressed by Berger and Kostelecky [15]. They have shown that a supersymmetric matter model in the presence of a Lorentz violating term could be achieved with success. Following a different approach (starting from the degrees of freedom of the gauge sector), the work of [11] has recently built up a supersymmetric minimal extension of the Carroll–Field–Jackiw model, obtaining also a non-polynomial extension compatible with $N = 1$ SUSY. On other hand, the issue of the SSB was first addressed in [10], where the spectrum was thoroughly discussed and electrically charged vortices were found.

This broad interest on the Carroll–Field–Jackiw model has triggered the investigation of a similar model in a lower dimensional context. In this way, the dimensional reduction (to 1 + 2 dimensions) of the Carroll–Field–Jackiw model [7] was successfully realized [19], resulting in a planar theory composed of a Maxwell–Chern–Simons gauge field (A_{μ}), a massless scalar field (φ), and a coupling term, $\varphi\epsilon_{\mu\nu\kappa}v^{\mu}\partial^{\nu}A^{\kappa}$, responsible for the Lorentz violation. The reduced model has been shown to preserve causality, stability and unitarity (in the gauge sector) both for a space- and timelike backgrounds (without any restriction) [19], which bypasses the lack of positivity and causality manifest in the 4-dimensional original model. Such a result has put in evidence that this reduced model can undergo a consistent quantization program (for both timelike and spacelike backgrounds). Another interesting issue refers to the classical electrodynamics concerning this planar Lagrangian, investigated initially at the level of the equations of motion taken at the static limit. Preliminary results [20] show that a purely timelike background induces the behavior of a massless electrodynamics (in the electric sector), while a pure spacelike background appears as a factor of strong anisotropy promotion. The study of the scalar potential (A_0) solutions reveals the existence of a region where it is negative, which favors the attainment of an electron–electron attractive potential, a fact of relevance in connection with condensed matter physics and recently confirmed at least for a purely timelike background [22].

In this work, we aim at constructing and investigating a planar Lorentz violating model endowed with the Higgs sector. An extension of the Carroll–Field–Jackiw model in $(1 + 3)$ dimensions, including a scalar sector that yields spontaneous symmetry breaking (Higgs sector) [10], was recently developed and analyzed, providing an abelian Higgs gauge model with violation of Lorentz symmetry. The planar counterpart of this abelian Higgs model can be obtained by means of a dimensional reduction (to $1+2$ dimensions). The main motivation to study this kind of model is twofold:
(i) the relevance of considering a Lorentz violating planar model with spontaneous $U(1)$-symmetry breaking, which opens up the possibility of analyzing the physical consistency of a Lorentz violating theoretical framework endowed with a Higgs sector in $(1 + 2)$ dimensions;
(ii) the need of obtaining screened solutions, which is associated with condensed matter systems, where one usually works with short range solutions. The presence of the Higgs sector makes feasible promising investigations on vortex configurations [16], which may be of interest in connection with anisotropic condensed matter systems.

In the present work, however, we really focus attention on the first point: starting from the abelian Higgs model developed in [10], we perform its dimensional reduction, having as an outcome a planar quantum electrodynamics (QED$_3$) described by a Maxwell–Chern–Simons gauge field, A_{μ}, by a massive Klein–Gordon field, φ, and by the scalar sector (ϕ) minimally coupled to the gauge field, from which the Higgs sector stems. The φ field also works out as the coupling constant in the term that mixes the gauge field to the fixed 3-vector, v^{μ}. A fourth-order scalar potential, V, then induces a spontaneous symmetry breaking, which yields the appearance of the Higgs scalar and a Proca mass component to the gauge field. Having established the new planar Lagrangian, one then devotes some effort for the evaluation of the propagators of the gauge and scalar fields, which requires the definition of a closed algebra composed of eleven spin operators. Afterwards, the physical consistency of this model is investigated, with causality, stability and unitarity being analyzed at the classical level. Despite the presence of non-causal modes $\left(k^2 < 0\right)$ coming from the dispersion relations, the evaluation and analysis of the group and front velocities is taken as a suitable criterion for assuring the causality. Here, as it occurs in the reduced version [19] of the Maxwell–Carroll–Field–Jackiw model, the model appears to be totally stable, causal and unitary for both time- and spacelike backgrounds, at the classical level, bypassing the absence of stability and causality exhibited by the original CFJ model. Once the unitarity is guaranteed, this model may undergo a consistent quantization program, which is an important requirement for the application of this model to describe physical systems.

This work is outlined as follows. In Sect. 2, we first perform the dimensional reduction of the abelian Higgs Carroll–Field–Jackiw model, obtaining the corresponding Lorentz violating planar model. Afterwards, spontaneous symmetry breaking is considered and the propagators of the gauge and scalar fields are evaluated. Knowledge of the propagators allows the investigation to be made of the physical consistency of this model. In Sect. 3, the stability and the causal structure of the model are analyzed, starting from the dispersion relations extracted from the propagators. In Sect. 4, the unitarity is suitably analyzed via the method of the residues (evaluated at the poles of the propagators) of the current–current saturated propagator. Finally, in Sect. 5, we present our concluding remarks.

2 The dimensionally reduced model

The starting point is a typical scalar electrodynamics, defined in $(1+3)$ dimensions, endowed with the Carroll–Field–Jackiw term, as written in [10]:

$$\begin{aligned}\mathcal{L}_{1+3} = &-\frac{1}{4}F_{\hat{\mu}\hat{\nu}}F^{\hat{\mu}\hat{\nu}} + \frac{1}{4}\varepsilon^{\hat{\mu}\hat{\nu}\hat{\kappa}\hat{\lambda}}v_{\hat{\mu}}A_{\hat{\nu}}F_{\hat{\kappa}\hat{\lambda}}\\ &+ (D^{\hat{\mu}}\phi)^* D_{\hat{\mu}}\phi - V(\phi^*\phi) + A_{\hat{\nu}}J^{\hat{\nu}},\end{aligned} \tag{1}$$

where the $\hat{\mu}$ runs from 0 to 3, $D_{\hat{\mu}} = (\partial_{\hat{\mu}} + \mathrm{i}eA_{\hat{\mu}})$ is the covariant derivative and $V(\phi^*\phi) = m^2\phi^*\phi + \delta(\phi^*\phi)^2$ represents the scalar potential responsible for spontaneous symmetry breaking ($m^2 < 0$ and $\delta > 0$). This model is gauge invariant but does not preserve the Lorentz and CTP symmetries.

In order to investigate this model in $(1+2)$ dimensions, it is necessary to perform its dimensional reduction, which consists effectively in adopting the following ansatz over any 4-vector:
(i) one keeps unaffected the time and also the first two space components;
(ii) one freezes the third space dimension by splitting it from the body of the new 3-vector, ascribing to it a scalar character; at the same time one requires that the new quantities (χ), defined in $(1+2)$ dimensions, do not depend on the third spacial dimension: $\partial_3\chi \longrightarrow 0$. Applying this prescription to the gauge 4-vector, $A^{\hat{\mu}}$, and the fixed external 4-vector, $v^{\hat{\mu}}$, one has

$$A^{\hat{\nu}} \longrightarrow (A^{\nu};\ \varphi), \tag{2}$$

$$v^{\hat{\mu}} \longrightarrow (v^{\mu};\ s), \tag{3}$$

where $A^{(3)} = \varphi$, $v^{(3)} = s$ are two scalars, and $\mu = 0, 1, 2$. Carrying out this prescription for (1), one then obtains

$$\begin{aligned}\mathcal{L}_{1+2} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{s}{2}\varepsilon_{\mu\nu\kappa}A^{\mu}\partial^{\nu}A^{\kappa} - \varphi\varepsilon_{\mu\nu\kappa}v^{\mu}\partial^{\nu}A^{\kappa}\\ &+\frac{1}{2}\partial_{\mu}\varphi\partial^{\mu}\varphi + (D^{\mu}\phi)^*D_{\mu}\phi - e^2\varphi^2\phi^*\phi\\ &-V(\phi^*\phi) + A_{\nu}J^{\nu} + \varphi J.\end{aligned} \tag{4}$$

The scalar, φ, endowed with dynamics, is a typical Klein–Gordon massless field, whereas s is a constant scalar (without dynamics), which acts as the Chern–Simons mass. The scalar field also appears as the coupling constant that links the fixed v^{μ} to the gauge sector of the model by means of the new term: $\varphi\varepsilon_{\mu\nu k}v^{\mu}\partial^{\nu}A^{k}$. In spite of being covariant in form, this kind of term breaks the Lorentz symmetry, since the 3-vector v^{μ} does not present dynamics. The presence of the Chern–Simons term in the Lagrangian (4), will also amount to the breakdown of the parity and time reversal symmetries.

In adopting the dimensional reduction prescription as specified above $(\partial_3\chi \longrightarrow 0)$, we can better clarify that the integration over the x_3-coordinate, taken as usually to be compact, will produce the length dimension that can be suitably absorbed into the field and coupling constant redefinitions so as to yield the right canonical dimensions for the fields and gauge coupling constant in $(1+2)$D. This means that the Lagrangian $\mathcal{L}_{1+2}$ naturally carries the right canonical dimensions in 3D once its corresponding 4-dimensional master action has been fixed up. The dimensional reduction procedure produces the right dimensional factor in such a way that the mass dimensions turn out to be the ordinary ones.

Concerning the gauge invariance, it is noteworthy to state that the reduced theory is gauge invariant under the reduction procedure (2) and (3). Indeed, the fact that all fields and the gauge parameter do not depend on the third spatial coordinate (x_3) guarantees that the scalar field, φ, is a gauge-invariant field in $(1+2)$D. On the other hand, the scalar s, identified with $v^{(3)}$, is a constant mass parameter; this shows that the term $\varepsilon_{\mu\nu k}A^{\mu}\partial^{\nu}A^{k}$ is a genuine Chern–Simons term, gauge invariant up to a surface term. So, the reduction prescription here implemented allows the gauge symmetry of the action in $(1+3)$D to survive in the planar regime. Therefore, both the actions in four and three space-time dimensions are gauge-invariant modulo surface terms.

According to the prescription of dimensional reduction here adopted, a comment is noteworthy: in the case the 4-dimensional background is purely spacelike and orthogonal to the $(1+2)$-dimensional subspace, that is, $v^{\hat{\mu}} = (0,0,0,v)$, there appears no sign of Lorentz violation in the reduced Lagrangian (4), once we are left with the genuine Chern–Simons topological mass term.

We now proceed to carry out the spontaneous symmetry breaking, that takes place when the scalar field exhibits a non-null vacuum expectation value: $\langle\phi\phi\rangle = -m^2/2\delta$. Adopting the parametrization $\phi = (\varkappa +$

$\eta/\sqrt{2})\mathrm{e}^{\mathrm{i}\rho\eta/\sqrt{2}}$, we obtain (for $\rho = 0$)

$$\mathcal{L}_{1+2}^{\text{Broken}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{s}{2}\varepsilon_{\mu\nu\kappa}A^{\mu}\partial^{\nu}A^{\kappa} - \varphi\varepsilon_{\mu\nu\kappa}v^{\mu}\partial^{\nu}A^{\kappa} + \frac{1}{2}\partial_{\mu}\varphi\partial^{\mu}\varphi - e^2\varkappa^2\varphi^2 + e^2\varkappa^2 A_{\mu}A^{\mu} + \frac{1}{2}\partial_{\mu}\eta\partial^{\mu}\eta + \frac{2}{\sqrt{2}}e^2\varkappa\eta A_{\mu}A^{\mu} + \frac{e^2}{2}\eta^2 A_{\mu}A^{\mu} + m^2(\varkappa + \eta/\sqrt{2})^2 + \delta(\varkappa + \eta/\sqrt{2})^4. \quad (5)$$

Retaining only tree-level terms, we obtain the action in an explicitly quadratic form,

$$\Sigma_{1+2} = \int \mathrm{d}^3x \frac{1}{2}\{A^{\mu}[Z_{\mu\nu}]A^{\nu} - \varphi(\Box + M_A^2)\varphi - \varphi\left[\epsilon_{\mu\alpha\nu}v^{\mu}\partial^{\alpha}\right]A^{\nu} + A^{\mu}\left[\epsilon_{\nu\alpha\mu}v^{\nu}\partial^{\alpha}\right]\varphi\}, \quad (6)$$

where the mass of the scalar field is the same as the Proca mass ($M_A^2 = 2e^2\varkappa^2$). Here, the mass dimension of the physical parameters and tensors are $[A^{\mu}] = [\varphi] = 1/2$, $[v^{\mu}] = [s] = 1, [T_{\mu}] = [Z_{\mu\nu}] = 2$. The action (6) can also be read in a matrix form:

$$\Sigma_{1+2} = \int \mathrm{d}^3x \frac{1}{2}\begin{pmatrix} A^{\mu} & \varphi \end{pmatrix}\begin{bmatrix} Z_{\mu\nu} & T_{\mu} \\ -T_{\nu} & -(\Box + M_A^2) \end{bmatrix}\begin{pmatrix} A^{\nu} \\ \varphi \end{pmatrix}. \quad (7)$$

Now, we define the operators we shall be dealing with:

$$Z_{\mu\nu} = \Box\theta_{\mu\nu} + sS_{\mu\nu} + M_A^2 g_{\mu\nu}, \quad T_{\mu} = S_{\nu\mu}v^{\nu}, \quad (8)$$

$$S_{\mu\nu} = \varepsilon_{\mu\kappa\nu}\partial^{\kappa}, \quad \theta_{\mu\nu} = \eta_{\mu\nu} - \omega_{\mu\nu}, \quad \omega_{\mu\nu} = \frac{\partial_{\mu}\partial_{\nu}}{\Box}, \quad (9)$$

where $\theta_{\mu\nu}$ and $\omega_{\mu\nu}$ are respectively the dimensionless transverse and longitudinal projectors.

The propagators of the gauge and scalar fields are given by the inverse of the square matrix, Q, associated with the action (7). The propagator matrix, Δ, is then written as

$$\Delta = Q^{-1} = \frac{-1}{(\Box + M_A^2)Z_{\mu\nu} - T_{\mu}T_{\nu}}\begin{bmatrix} -(\Box + M_A^2) & T_{\nu} \\ -T_{\mu} & Z_{\mu\nu} \end{bmatrix}, \quad (10)$$

whose components are given by

$$(\Delta_{11})^{\mu\nu} = (\Box + M_A^2)\left[+Z_{\mu\nu}(\Box + M_A^2) - T_{\mu}T_{\nu}\right]^{-1}, \quad (11)$$

$$(\Delta_{22}) = -Z_{\mu\nu}\left[Z_{\mu\nu}(\Box + M_A^2) - T_{\mu}T_{\nu}\right]^{-1}, \quad (12)$$

$$(\Delta_{12})^{\mu} = -T_{\nu}\left[Z_{\mu\nu}(\Box + M_A^2) - T_{\mu}T_{\nu}\right]^{-1}, \quad (13)$$

$$(\Delta_{21})^{\nu} = T_{\mu}\left[Z_{\mu\nu}(\Box + M_A^2) - T_{\mu}T_{\nu}\right]^{-1}. \quad (14)$$

The terms Δ_{11}, Δ_{22} correspond to the propagators of the gauge and scalar fields, while the terms Δ_{12}, Δ_{21} are the mixed propagators $\langle\varphi A_{\mu}\rangle$, $\langle A_{\mu}\varphi\rangle$, which describe a scalar mediator turning into a gauge mediator and vice versa. In order to explicitly obtain these propagators, it is necessary to invert the matrix components individually. For this purpose, one needs to create some new operators, in such a way that a closed operator algebra can be defined. In this sense, we define the following tensor operators:

$$Q_{\mu\nu} = v_{\mu}T_{\nu}, \quad \Lambda_{\mu\nu} = v_{\mu}v_{\nu}, \quad \Sigma_{\mu\nu} = v_{\mu}\partial_{\nu}, \quad \Phi_{\mu\nu} = T_{\mu}\partial_{\nu}, \quad (15)$$

which fulfill some useful relations:

$$S_{\mu\nu}T^{\nu}T^{\alpha} = \Box v_{\mu}T^{\alpha} - \lambda T^{\alpha}\partial_{\mu} = \Box Q_{\mu}{}^{\alpha} - \lambda\Phi^{\alpha}{}_{\mu}, \quad (16)$$

$$Q_{\mu\nu}Q^{\alpha\nu} = T^2 v^{\alpha}v_{\mu} = T^2\Lambda^{\alpha}{}_{\mu}, \quad Q_{\mu\nu}\Phi^{\nu\alpha} = T^2 v_{\mu}\partial^{\alpha} = T^2\Sigma_{\mu}{}^{\alpha}, \quad (17)$$

$$\lambda \equiv \Sigma_{\mu}^{\ \mu} = v_{\mu}\partial^{\mu}, \quad T^2 = T_{\alpha}T^{\alpha} = (v^2\Box - \lambda^2). \quad (18)$$

Their mass dimensions are $[\Lambda_{\mu\nu}] = 2$, $[Q_{\mu\nu}] = 3$, $[\Sigma_{\mu\nu}] = 2$, $[\Phi_{\mu\nu}] = 3$.

The inversion of Δ_{11} is realized following the traditional prescription, $\left(\Delta_{11}^{-1}\right)_{\mu\nu}(\Delta_{11})^{\nu\alpha} = \delta_{\mu}^{\alpha}$, where the operator $(\Delta_{11})^{\nu\alpha}$ is the most general tensor operator composed of rank 2 combinations of the one-forms $T_{\mu}, v_{\mu}, \partial_{\alpha}$. In this sense, the operators $Q_{\mu\nu}, Q_{\nu\mu}$, $\Sigma_{\mu\nu}, \Sigma_{\nu\mu}, \Phi_{\mu\nu}, \Phi_{\nu\mu}$ must all be considered, leading to a linear combination of eleven terms:

$$(\Delta_{11})^{\nu\alpha} = a_1\theta^{\nu\alpha} + a_2\omega^{\nu\alpha} + a_3 S^{\nu\alpha} + a_4\Lambda^{\nu\alpha} + a_5 T^{\nu}T^{\alpha} + a_6 Q^{\nu\alpha} + a_7 Q^{\alpha\nu} + a_8\Sigma^{\nu\alpha} + a_9 Q^{\alpha\nu} + a_{10}\Phi^{\nu\alpha} + a_{11}\Phi^{\alpha\nu}. \quad (19)$$

The closure of the operator algebra involving these operators is contained in Table I of [19], whose application leads to the following propagator of the gauge field:

$$(\Delta_{11})^{\mu\nu} = \frac{(\Box + M_A^2)}{\boxplus}\theta^{\mu\nu} + \frac{(\Box + M_A^2)\boxtimes\boxplus - \lambda^2 s^2 M_A^2\Box}{M_A^2(\Box + M_A^2)\boxtimes\boxplus}\omega^{\mu\nu} - \frac{s}{\boxplus}S^{\mu\nu} - \frac{s^2\Box^2}{(\Box + M_A^2)\boxtimes\boxplus}\Lambda^{\mu\nu} + \frac{(\Box + M_A^2)}{\boxtimes\boxplus}T^{\mu}T^{\nu} - \frac{s\Box}{\boxtimes\boxplus}\left[Q^{\mu\nu} - Q^{\nu\mu}\right] + \frac{\lambda s^2\Box}{(\Box + M_A^2)\boxtimes\boxplus}\left[\Sigma^{\mu\nu} + \Sigma^{\nu\mu}\right] - \frac{s\lambda}{\boxtimes\boxplus}\left[\Phi^{\mu\nu} - \Phi^{\nu\mu}\right], \quad (20)$$

where $\boxtimes = \left[(\Box + M_A^2)^2 - T^2 + s^2\Box\right], \boxplus = (\Box + M_A^2)^2 + s^2\Box$.

According to (11)–(14), the propagators $(\Delta_{12})^{\alpha}$ and $(\Delta_{21})^{\alpha}$ can be written in terms of the Δ_{11}-gauge propagator,

$$(\Delta_{12})^{\alpha} = -\frac{T_{\mu}}{(\Box + M_A^2)}(\Delta_{11})^{\mu\alpha}, \quad (\Delta_{21})^{\alpha} = \frac{T_{\mu}}{(\Box + M_A^2)}(\Delta_{11})^{\alpha\mu}, \quad (21)$$

which leads to the following propagator expressions:

$$(\Delta_{12})^{\alpha} = \frac{-1}{(\Box + M_A^2)\boxtimes}\left[(\Box + M_A^2)T^{\alpha} + s\Box v^{\alpha} - s\lambda\partial^{\alpha}\right], \quad (22)$$

$$(\Delta_{21})^{\alpha} = \frac{1}{(\Box + M_A^2)\boxtimes}\left[(\Box + M_A^2)T^{\alpha} - s\Box v^{\alpha} + s\lambda\partial^{\alpha}\right]. \quad (23)$$

As for the scalar field propagator, it can be put in the tensor form:

$$(\Delta_{22}) = -\left[(\Box + M_A^2) - T_{\mu}(Z_{\mu\nu})^{-1}T_{\nu}\right]^{-1}, \quad (24)$$

which can be easily solved by taking the inverse of the tensor $Z_{\mu\nu}$,

$$(Z_{\mu\nu})^{-1} = \frac{(\Box + M_A^2)}{\boxplus}\theta^{\mu\nu} - \frac{s}{\boxplus}S^{\mu\nu} + \frac{1}{M_A^2}\omega^{\mu\nu}. \quad (25)$$

Making use of the following outcome: $T_{\mu}(Z^{-1})^{\mu\nu}T_{\nu} = (\Box + M_A^2)T^2/\boxplus$, a simple scalar propagator arises:

$$(\Delta_{22}) = -\frac{\boxplus}{\boxtimes(\Box + M_A^2)}.$$

Now, we can write the propagators here obtained in momentum-space. The photon propagator takes on its final form:

$$\begin{aligned}
&\langle A^{\mu}(k)\, A^{\nu}(k)\rangle \\
&= \mathrm{i}\left\{-\frac{(k^2 - M_A^2)}{\boxplus(k)}\theta^{\mu\nu}\right. \\
&+\frac{(k^2 - M_A^2)\boxtimes\boxplus - \lambda^2 s^2 M_A^2 k^2}{M_A^2(k^2 - M_A^2)\boxtimes(k)\boxplus(k)}\omega^{\mu\nu} \\
&-\frac{s}{\boxplus}S^{\mu\nu} + \frac{s^2k^4}{(k^2 - M_A^2)\boxtimes(k)\boxplus(k)}\Lambda^{\mu\nu} \\
&-\frac{(k^2 - M_A^2)}{\boxtimes(k)\boxplus(k)}T^{\mu}T^{\nu} + \frac{sk^2}{\boxtimes(k)\boxplus(k)}\left[Q^{\mu\nu} - Q^{\nu\mu}\right] \\
&+\frac{\mathrm{i}(v.k)s^2k^2}{(k^2 - M_A^2)\boxtimes(k)\boxplus(k)}\left[\Sigma^{\mu\nu} + \Sigma^{\nu\mu}\right] \\
&\left.-\frac{\mathrm{i}s(v.k)}{\boxtimes(k)\boxplus(k)}\left[\Phi^{\mu\nu} - \Phi^{\nu\mu}\right]\right\}, \qquad (26)
\end{aligned}$$

while the scalar and the mixed propagators read

$$\langle\varphi\varphi\rangle = \mathrm{i}\frac{\boxplus(k)}{\boxtimes(k)(k^2 - M_A^2)}, \quad (27)$$

$$\begin{aligned}\langle A^{\alpha}\varphi\rangle = &\frac{-\mathrm{i}}{(k^2 - M_A^2)\boxtimes(k)} \quad (28)\\
&\times\left[(k^2 - M_A^2)T^{\alpha} + sk^2v^{\alpha} + s(v\cdot k)k^{\alpha}\right],\end{aligned}$$

$$\begin{aligned}\langle\varphi A^{\alpha}\rangle = &\frac{\mathrm{i}}{(k^2 - M_A^2)\boxtimes(k)} \quad (29)\\
&\times\left[(k^2 - M_A^2)T^{\alpha} - sk^2v^{\alpha} + s(v\cdot k)k^{\alpha}\right],\end{aligned}$$

where $\boxtimes(k) = k^4 - \left(2M_A^2 + s^2 - v\cdot v\right)k^2 + M_A^4 - (v\cdot k)^2$, $\boxplus(k) = (k^2 - M_A^2)^2 - s^2k^2$.

Since we are committed to the calculation of physical quantities such as the mass spectrum and the residues of the propagators at their poles, we take the viewpoint of working in the unitary gauge. Local $U(1)$-symmetry has been spontaneously broken, so that we could have also chosen to adopt the R_{ξ}-type gauge, for which the would-be Goldstone scalar propagates (its pole is however gauge-dependent) and the longitudinal part of the gauge-field propagator displays the same gauge-dependent pole. However, this gauge is more convenient for the study of more formal aspects, like renormalizability, for example. To get information on the mass spectrum and on the physical character of the propagator poles, the choice of the unitary gauge seems to be more natural. It is the gauge symmetry, even though spontaneously broken, that allows us to adopt either choice; once at the level of the S-matrix the results will be perfectly equivalent.

3 Causality and stability analysis

Despite Lorentz symmetry being a cornerstone of field theory, Lorentz violating theoretical models may be acceptable once there occurs preservation of two physical essential properties: causality and stability (energy positivity). The poles of the propagators can be taken as a suitable starting point to get information about causality, stability and unitarity of the correlated model. The causality analysis, at tree level, is related to the sign of the propagator poles, given in terms of k^2, in such a way that one must have $k^2 \geq 0$ in order to preserve the causality (preventing the existence of tachyons). The families of poles at k^2 coming from the propagators expressions are given below:

$$k^2 = M_A^2; \;\; \boxplus(k) = 0\;; \;\; \boxtimes(k) = 0; \quad (30)$$

from which we extract the dispersion relations associated with each one. In the case of $k^2 = M_A^2$, we obtain a very simple dispersion relation, $k_0^2 = M_A^2 + \mathbf{k}^2$, which obviously establishes both a causal and stable mode.

Concerning the equation $\boxplus(k) = 0$, we attain background-independent roots:

$$k_{\pm}^2 = M_A^2 + \frac{s^2}{2} \pm \frac{|s|}{2}\sqrt{s^2 + 4M_A^2}. \quad (31)$$

The causality is preserved at these poles, since we have $k_{\pm}^2 > 0$. The stability of these modes is also assured.

As for the poles of $\boxtimes(k) = 0$, we obtain

$$\begin{aligned}k_{\pm}^2 = &M_A^2 + \frac{s^2}{2} - \frac{v\cdot v}{2} \quad (32)\\
&\pm\frac{1}{2}\sqrt{(s^2 - v\cdot v)(s^2 - v\cdot v + 4M_A^2) + 4(v\cdot k)^2}.\end{aligned}$$

In the case of a purely timelike background, $v^{\mu} = (v_0, \mathbf{0})$, these poles assume the following form:

$$k_{\pm}^2 = M_A^2 + s^2/2 \pm \sqrt{s^4/4 + M_A^2s^2 + v_0^2\mathbf{k}^2}, \quad (33)$$

from which we note that the pole k_+^2 is always causal and stable whereas the pole k_-^2, beyond to be non-causal ($k_-^2 < 0$), seems to be non-stable. Hence, the first analysis of relevance refers to the stability (positivity of the energy) of the mode k_-^2. A simple investigation reveals that the expression for the energy, $k_{0-}^2 = M_A^2 + s^2/2 + \mathbf{k}^2 \pm \sqrt{s^4/4 + M_A^2 s^2 + v_0^2\mathbf{k}^2}$, is always positive for any value of $\mathbf{k}^2$ whenever the single condition $s^2 > v_0^2$ is fulfilled. Once the stability is assured, it turns out to be feasible to show that the non-causal character of this last pole ($k_-^2 < 0$) is not decisive as regards spoiling the causality of the model. In order to do it, one takes as essential point the evaluation of the group and the front velocities associated with the pole k_-^2. Adopting $k^\mu = (k_0, 0, k_2)$, the group velocity ($v_\mathrm{g} = \mathrm{d}k_{0-}/\mathrm{d}k_2$) as a result equals

$$v_\mathrm{g} = \frac{k_2}{k_{0-}} \frac{\sqrt{s^4/4 + M_A^2 s^2 + v_0^2 k_2^2} - v_0^2/2}{\sqrt{s^4/4 + M_A^2 s^2 + v_0^2 k_2^2}}. \quad (34)$$

Such a velocity is always less than 1, once the energy expression for k_{0-} does not possess any pole (it is positive-definite for any value of k^2). In the limit $k_2 \to \infty$, one has $v_\mathrm{g} = 1$. From the phase velocity ($v_\mathrm{ph} = k_{0-}/k_2$), one can obtain the front velocity ($v_\mathrm{f} = \lim_{k\to\infty} |v_\mathrm{ph}|$), which stands for a sensitive factor for signal propagation [17, 18]. Considering (33), one easily notes that it yields a unitary front velocity ($v_\mathrm{f} = 1$) in the limit $k_2 \to \infty$, which regarded jointly with $v_\mathrm{g} \leq 1$ constitutes a suitable criterion to assure causality at classical level.

For a purely spacelike background, $v^\mu = (0, \mathbf{v})$, (32) reads

$$k_\pm^2 = M_A^2 + s^2/2 + \mathbf{v}^2/2 \pm \frac{1}{2}\sqrt{(s^2 + \mathbf{v}^2)(s^2 + \mathbf{v}^2 + 4M_A^2) + 4(\mathbf{v}.\mathbf{k})^2}. \quad (35)$$

In this case, we have the same behavior as in the purely timelike situation, that is, the pole k_+^2 is always causal and stable, whereas the pole k_-^2 is non-causal ($k_-^2 < 0$). Now, one can show that the stability of this mode can be assured $\left(k_{0-}^2 > 0\right)$ without any restriction over the parameters. Adopting $k^\mu = (k_0, 0, k_2)$, the group velocity ($v_\mathrm{g} = \mathrm{d}k_{0-}/\mathrm{d}k_2$) is then given as follows:

$$v_\mathrm{g} = \frac{k_2}{k_{0-}} \frac{\sqrt{(s^2 + \mathbf{v}^2)(s^2 + \mathbf{v}^2 + 4M_A^2) + 4\mathbf{v}_2^2 k_2^2} - \mathbf{v}_2^2}{\sqrt{(s^2 + \mathbf{v}^2)(s^2 + \mathbf{v}^2 + 4M_A^2) + 4\mathbf{v}_2^2 k_2^2}}. \quad (36)$$

This expression implies that $v_\mathrm{g} < 1$ for any value of k_2 and $v_\mathrm{g} = 1$ in the limit $k_2 \to \infty$. Analogously, it may be shown that the front velocity is unitary ($v_\mathrm{f} = 1$), a sufficient condition to protect the spectrum of the model from the presence of non-causal modes and to assure the causality of physical signals. Therefore, despite the presence of non-causal poles $\left(k_-^2 < 0\right)$ in both time- and spacelike cases, the conditions $v_\mathrm{g} < 1$ and $v_\mathrm{f} = 1$ exclude the appearance of tachyons.

4 Unitarity

The unitarity analysis of the reduced model at tree level is here carried out through the saturation of the propagators with external currents, which must be implemented both for the scalar (J) and gauge (J^μ) currents, once the model presents these two sectors. In such a way, we write individually the two saturated propagators (SP) in the following form:

$$\mathrm{SP}_{\langle A_\mu A_\nu \rangle} = J^{*\mu} \langle A_\mu(k) A_\nu(k) \rangle \; J^\nu, \quad (37)$$

$$\mathrm{SP}_{\langle \varphi\varphi \rangle} = J^* \langle \varphi\varphi \rangle J. \quad (38)$$

While the gauge current (J^μ) satisfies the conservation law ($\partial_\mu J^\mu = 0$), the scalar current (J) does not fulfill any constraint. In the context of this method, the unitarity analysis is assured whenever the imaginary part of the residues of the SP at the poles of each propagator is positive.

4.1 Scalar sector

The unitarity analysis of the scalar sector is performed by means of (38), or more explicitly:

$$\mathrm{SP}_{\langle \varphi\varphi \rangle} = J^* \frac{\mathrm{i} \boxplus (k)}{\boxtimes(k)(k^2 - M_A^2)} J. \quad (39)$$

This expression presents three poles: M_A^2, and k_+^2, k_-^2 (the roots of $\boxtimes(k) = 0$). In the purely timelike case, $v^\mu = (v_0, \mathbf{0})$, the poles $k_\pm^2$ are exactly the ones given by (33). The residues of $\mathrm{SP}_{\langle \varphi\varphi \rangle}$, evaluated at these three poles, are positive-definite, and in such a way the unitarity of the scalar sector, in the timelike case, is completely assured.

In the purely spacelike case, $v^\mu = (0, \mathbf{v})$, the poles of (39) are M_A^2 and the ones given by (35). The residues of $\mathrm{SP}_{\langle \varphi\varphi \rangle}$, carried out at these three poles, provide us with a positive-definite imaginary part, so that the unitarity in the spacelike case is generically preserved. So, we conclude that the unitarity of the scalar sector is ensured without any restrictions.

4.2 Gauge field

As for the gauge field, the continuity equation, $k_\mu J^\mu = 0$, reduces to six the number of terms of the photon propagator that contributes to the evaluation of the saturated propagator:

$$\begin{aligned}\mathrm{SP} &= J_\mu^*(k) \left\{ \frac{\mathrm{i}}{D} \left[(\Box + M_A^2)^2 \boxtimes g^{\mu\nu} - s(\Box + M_A^2) \boxtimes S^{\mu\nu} \right.\right. \\ &\quad - s^2 \Box^2 \Lambda^{\mu\nu} + (\Box + M_A^2)^2 T^\mu T^\nu \\ &\quad \left.\left. - \; s\Box(\Box + M_A^2)(Q^{\mu\nu} - Q^{\nu\mu}) \right] \right\} J_\nu(k), \quad (40)\end{aligned}$$

where $D = (\Box + M_A^2) \boxtimes \boxplus$. In this case, the current components exhibit the form $J^\mu = \left(j^0, 0, \frac{k_0}{k_2} j^{(0)}\right)$ whenever one adopts as momentum $k^\mu = (k_0, 0, k_2)$. Writing this expression in momentum-space, one obtains

$$\mathrm{SP} = J^{*\mu}(k)\{B_{\mu\nu}\}J^\nu(k), \tag{41}$$

where $D = -(k^2 - M_A^2)\boxtimes(k)\boxplus(k)$, $\boxtimes(k) = k^4 - \left(2M_A^2 + s^2 - v\cdot v\right)k^2 + M_A^4 - (v\cdot k)^2$, $\boxplus(k) = (k^2 - M_A^2)^2 - s^2k^2$.

4.2.1 Timelike case

For a purely timelike 3-vector, $v^\mu = (v_0, \mathbf{0})$, $k^\mu = (k_0, 0, k_2)$, the tensor $B_{\mu\nu}$ is given as follows:

$$B_{\mu\nu}(k) = \frac{\mathrm{i}}{D(k)} \times \begin{bmatrix} C^4\boxtimes - s^2v_0^2k^4 & -\mathrm{i}sk^{(2)}C^2[\boxtimes - v_0^2k^2] & \mathrm{i}sC^2k^{(1)}(\boxtimes - v_0^2k^2) \\ \mathrm{i}sk^{(2)}C^2[\boxtimes - v_0^2k^2] & -C^4[\boxtimes + v_0^2k_2^2] & -C^2[\mathrm{i}s\boxtimes k_0 - C^2v_0^2k^{(1)}k^{(2)}] \\ -\mathrm{i}sC^2k^{(1)}(\boxtimes - v_0^2k^2) & C^2[\mathrm{i}s\boxtimes k_0 + C^2v_0^2k^{(1)}k^{(2)}] & -C^4[\boxtimes + v_0^2k_1^2] \end{bmatrix}, \tag{42}$$

where we used the short notation $C^2 = (k^2 - M_A^2)$.

We start by performing a unitarity analysis for the first pole, $k^2 = M_A^2$, for which the residue of the matrix $B_{\mu\nu}$ can be reduced to a very simple form:

$$B_{\mu\nu}(M_A^2) = \mathrm{i}\frac{M_A^2v_0^2}{[s^2M_A^2 + v_0^2k_2^2]}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{43}$$

which implies a positive saturation (SP > 0), and preservation of unitarity.

For the poles of $\boxtimes(k) = 0$, given by (33), we obtain the following residue matrix:

$$B_{\mu\nu}(k_\pm^2) = \mathrm{i}R_\pm v_0^2 \times \begin{bmatrix} s^2k_\pm^4 & -\mathrm{i}sk_\pm^2(k_\pm^2 - M_A^2)k^{(2)} & 0 \\ \mathrm{i}sk_\pm^2(k_\pm^2 - M_A^2)k^{(2)} & (k_\pm^2 - M_A^2)^2k_2^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{44}$$

where $R_\pm$ is the residue of $1/D(k)$ evaluated at $k_\pm^2$, namely

$$R_\pm = 2v_0^2\mathbf{k}^2\left(s^2/2 \pm \sqrt{s^4/4 + 4M_A^2s^2 + 4v_0^2\mathbf{k}^2}\right) \times \left(\pm\sqrt{s^4/4 + 4M_A^2s^2 + 4v_0^2\mathbf{k}^2}\right),$$

which implies $(R_\pm) > 0$. The eigenvalues of the matrix above are $\lambda_1 = 0; \lambda_2 = 0; \lambda_3 = s^2k_\pm^4 + k_2^2(k_\pm^4 - 2M_A^2k_\pm^2 + M_A^4)$. Since λ_3 is a positive eigenvalue, the saturation results positive (SP > 0), and the unitarity is assured.

For the poles of $\boxplus(k) = 0$, given by (31), we obtain the following residue matrix:

$$B_{\mu\nu}|_{(k^2 = k_\pm^2)} = \mathrm{i}R_\pm \times \begin{bmatrix} C^4\boxtimes - s^2v_0^2k_\pm^4 & -\mathrm{i}sk^{(2)}C^2[\boxtimes - v_0^2k_\pm^2] & 0 \\ \mathrm{i}sk^{(2)}C^2[\boxtimes - v_0^2k_\pm^2] & 0 & -\mathrm{i}sC^2\boxtimes k_0 \\ 0 & \mathrm{i}sC^2\boxtimes k_0 & -C^4\boxtimes \end{bmatrix}, \tag{45}$$

where $C^2 = (k_\pm^2 - M_A^2), \boxtimes(k_\pm^2) = -v_0^2k_2^2$, and $R_\pm$ is the residue of $1/D(k)$ evaluated at $k_\pm^2$, so that $R_\pm > 0$. This matrix leads to a null saturation (SP $= 0$) whenever saturated with the external current $J^\mu = \left(j^0, 0, \frac{k_0}{k_2}j^{(0)}\right)$, which implies preservation of unitarity. The trivial saturation at these poles shows that the modes given by (31) are non-dynamical for the pure timelike background; therefore, they do not stand for a physical excitation.

4.2.2 Spacelike case

For a pure spacelike fixed vector, $v^\mu = (0, 0, V)$, $k^\mu = (k_0, 0, k_2)$, the rank 2 tensor $B_{\mu\nu}$ can be put in the following matrix form:

$$B_{\mu\nu}(k) = \frac{\mathrm{i}}{D(k)} \begin{bmatrix} C^4(\boxtimes - V^2k_1^2) & -\mathrm{i}C^2[s\boxtimes k^{(2)} + \mathrm{i}V^2k_0k^{(1)}] & \mathrm{i}sC^2(\boxtimes + V^2k^2)k^{(1)} \\ \mathrm{i}C^2[s\boxtimes k^{(2)} - \mathrm{i}V^2k_0k^{(1)}] & -C^4[\boxtimes + V^2k_0^2] & -\mathrm{i}sC^2[\boxtimes + V^2k^2]k_0 \\ -\mathrm{i}sC^2(\boxtimes + V^2k^2)k^{(1)} & \mathrm{i}sC^2[\boxtimes + V^2k^2]k_0 & -C^4\boxtimes - s^2V^2k^4 \end{bmatrix}. \tag{46}$$

First, we perform the unitarity analysis at the pole, $k^2 = M_A^2$, for which the residue of the matrix $B_{\mu\nu}$ can be simplified to a simple form:

$$B_{\mu\nu}(M_A^2) = \mathrm{i}\frac{s^2V^2M_A^4}{s^2[s^2M_A^2 + V^2M_A^2 + V^2k_2^2]}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{47}$$

which clearly implies a positive saturation (SP > 0) and preservation of unitarity.

For the poles of $\boxtimes(k) = 0$, given by (35), we obtain the following residue matrix:

$$B_{\mu\nu}(k_\pm^2) = -\mathrm{i}R_\pm V^2 \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & (k_\pm^2 - M_A^2)k_0^2 & \mathrm{i}s(k_\pm^2 - M_A^2)k_\pm^2k_0 \\ 0 & -\mathrm{i}s(k_\pm^2 - M_A^2)k_\pm^2k_0 & s^2k_\pm^4 \end{bmatrix}, \tag{48}$$

where $R_\pm$ is the residue of $1/D(k)$ evaluated at $k_\pm^2$, so that $R_\pm < 0$. The eigenvalues of the matrix above are $\lambda_1 = 0; \lambda_2 = 0; \lambda_3 = s^2k_\pm^4 + k_0^2(k_\pm^4 - 2M_A^2k_\pm^2 + M_A^4)$. Since λ_3 is a positive eigenvalue, the saturation as a result is found to be positive (SP > 0), and the unitarity is assured.

For the poles of $\boxplus(k) = 0$, given by (31), we obtain the following residue matrix:

$$B_{\mu\nu}|_{(k^2=k_\pm^2)} = \mathrm{i}R_\pm \times \begin{bmatrix} C^4\boxtimes & -\mathrm{i}sC^2\boxtimes k^{(2)} & 0 \\ \mathrm{i}sC^2\boxtimes k^{(2)} & 0 & \mathrm{i}sC^2(\boxtimes + V^2k_\pm^2)k_0 \\ 0 & -\mathrm{i}sC^2(\boxtimes + V^2k_\pm^2)k_0 & -C^4\boxtimes - s^2V^2k_\pm^4 \end{bmatrix}, \tag{49}$$

where $C^2 = (k_\pm^2 - M_A^2), \boxtimes(k_\pm^2) = -V^2k_0^2$, and $R_\pm$ is the residue of $1/D(k)$ evaluated at $k_\pm^2$, so that $R_\pm > 0$. This matrix, whenever saturated with the external current $J^\mu = \left(j^0, 0, \frac{k_0}{k_2}j^{(0)}\right)$, leads to a trivial saturation (SP = 0), which is compatible with unitarity requirements. The vanishing of SP at these poles indicates that the modes given by (31) are non-dynamical for the pure spacelike background too.

This is the whole lot of our investigations as far as causality and unitarity at tree level are concerned. We finish remarking that the reduced model preserves unitarity, for both space- and timelike backgrounds, without any restriction.

5 Concluding remarks

We have carried out the dimensional reduction to $(1+2$ dimensions) of an abelian Higgs gauge model with the Carroll–Field–Jackiw Lorentz violating term (defined in $1+3$ dimensions). One attains a planar model composed of a Maxwell–Chern–Simons–Proca gauge sector, a massive scalar sector and a mixing term that couples the gauge field to the fixed background. The propagators of this model are evaluated and the causality, stability and unitarity are analyzed. Concerning stability, it is entirely ensured whenever the auxiliary condition $s^2 > v_0^2$ is valid. Furthermore, we have shown that the overall model preserves causality and unitarity for both timelike and spacelike backgrounds, the same outcome as attained in [19]. This result encourages us to push forward our idea of applying the $(1+2)$D counterpart of the $(1+3)$ Lorentz broken models to discuss issues related to physical planar systems, once this model can be submitted to a consistent quantization scheme. Though fermions play a central role if we are committed to applications to low-dimensional condensed matter systems, we have not introduced them in our presentation. The reason is that the Lorentz breaking and its immediate consequence for the causality and unitarity are classically felt only by the charged scalars and gauge fields. The introduction of the fermions is the next natural step and what remains to be worked out is the influence (at the planar level) of the background vector, v^μ, in yielding Lorentz breaking terms in the fermionic sector. This matter is now under consideration.

A natural extension of the present work is the investigation of its classical equations of motion (for potentials and field strengths) and their corresponding solutions, in a similar way as it appears in [20]. Thus, the structure of the resulting electrodynamics associated with the planar Lagrangian (4) can be readily determined, at least in the static regime. Preliminary calculations reveal that the solutions for the field strengths and potentials have a very similar structure to the ones of the pure MCS–Proca electrodynamics. This issue is now under development [21]. A study of vortex configurations (for time- and spacelike backgrounds) was also carried out simultaneously to the analysis of the classical aspects alluded to here, revealing that this model is also endowed with stable vortex configurations [16].

Another point to be investigated concerns the evaluation of the electron–electron interaction in the context of this planar model. This matter may be analyzed in much the same way as adopted in [22], where one has evaluated the e^-e^- interaction potential for the case of the Lorentz violating MCS electrodynamics of [19]. It was then verified that the interaction potential may be attractive for some parameter values and exhibits a logarithmic potential near and far from the origin. In the case of the Lorentz violating MCS–Proca electrodynamics here developed, one expects the maintenance of the attractive character at the same time the resulting electron–electron potential is supposed to be totally screened due to the presence of the additional Proca mass parameter.

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